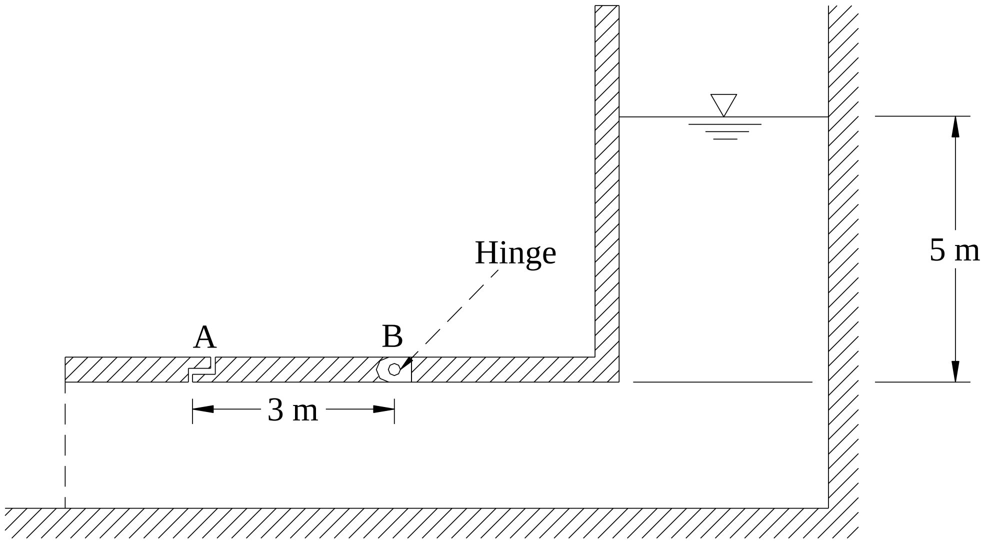
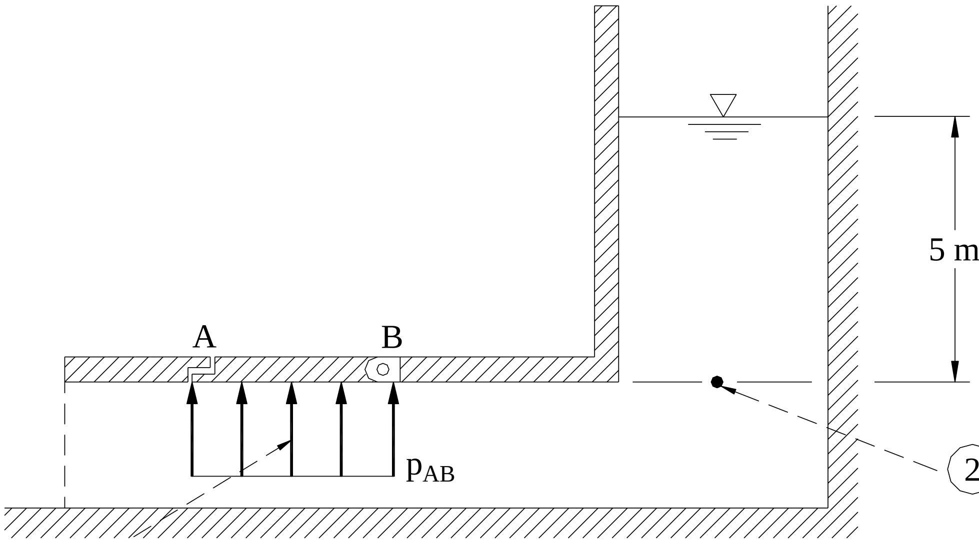
9.4 Resultants of Hydrostatic Pressure Forces

9.4 Resultants of Hydrostatic Pressure Forces Example 1, page 1 of 3

1. To prevent water pressure from pushing gate AB open, a small extension, or lip, is provided at A. If the gate is 4-m wide (measured perpendicular to the plane of the figure), determine the force acting on the lip. The density of water is $\rho = 10^3$ kg/m^3.





1 Since AB is horizontal, the pressure p_{AB} acting on it is uniform.

2 The pressure must be p_{AB} at all other points at the same elevation. Since these points lie 5 m below the free surface of the water,

$$p_{AB} = \rho g \times 5 \text{ m}$$

$$= (10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})$$

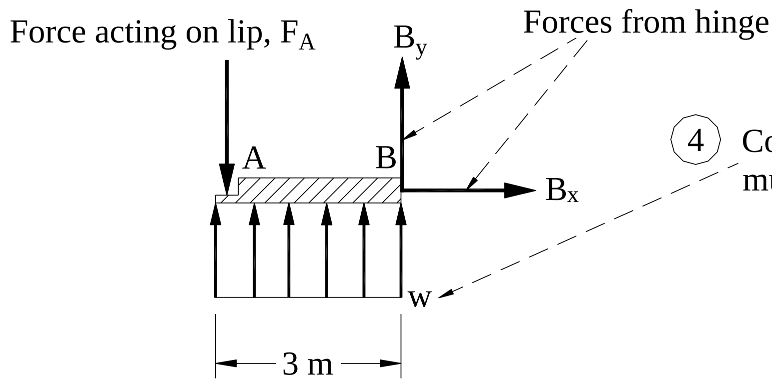
$$= 49.05 \times 10^3 \text{ (kg·m/s}^2\text{)/m}^2$$

$$= 49.05 \times 10^3 \text{ N/m}^2$$

$$= 49.05 \text{ kN/m}^2$$

$$= 49.05 \text{ kPa}$$

3 Free-body diagram of gate AB



4 Convert the pressure to a force per length by multiplying by the width:

$$w = p_{AB} \times \text{width of gate}$$

$$= (49.05 \text{ kPa}) \times 4 \text{ m}$$

$$= 196.20 \text{ kN/m}$$

Equilibrium equation for gate AB

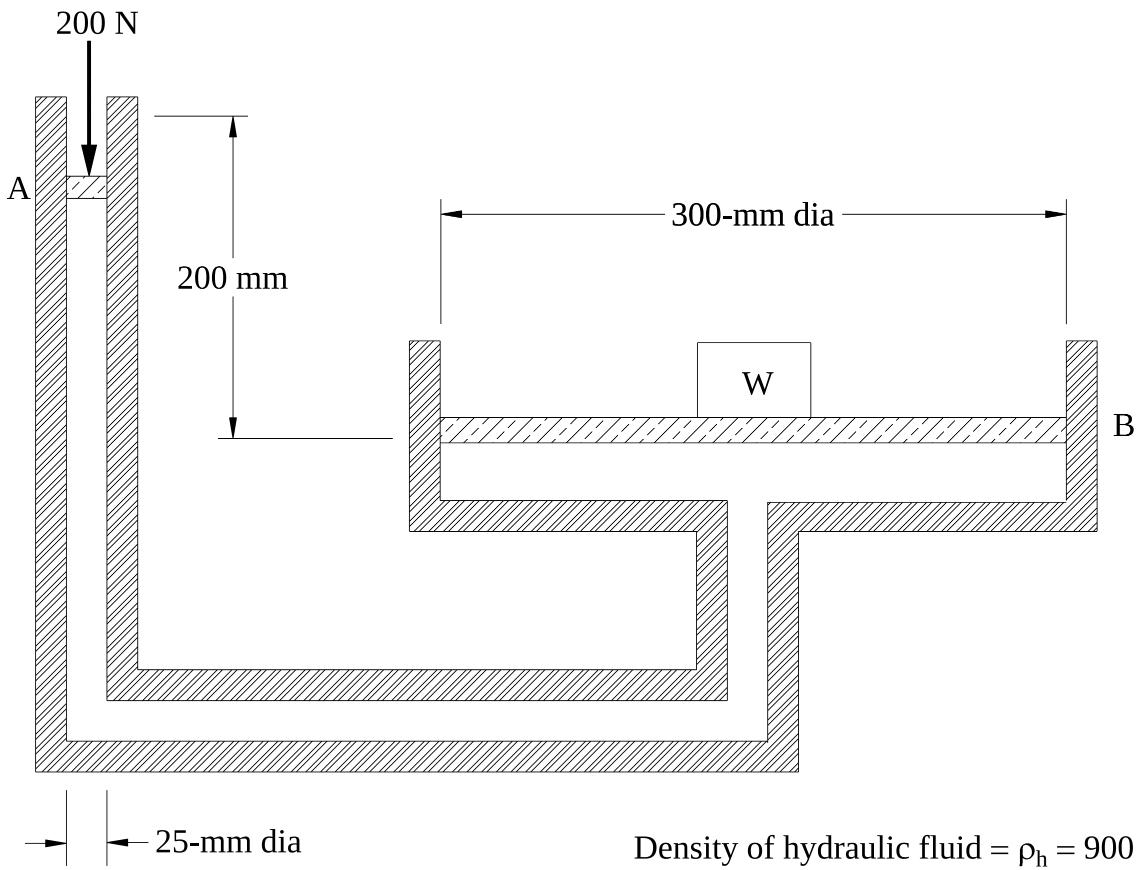
$$\circlearrowleft + \Sigma M_B = F_A(3 \text{ m}) - \underbrace{(196.20 \text{ kN/m})(3 \text{ m})}_{=\text{ resultant of } w}(3 \text{ m}/2) = 0$$

Moment arm of pressure resultant

Solving gives

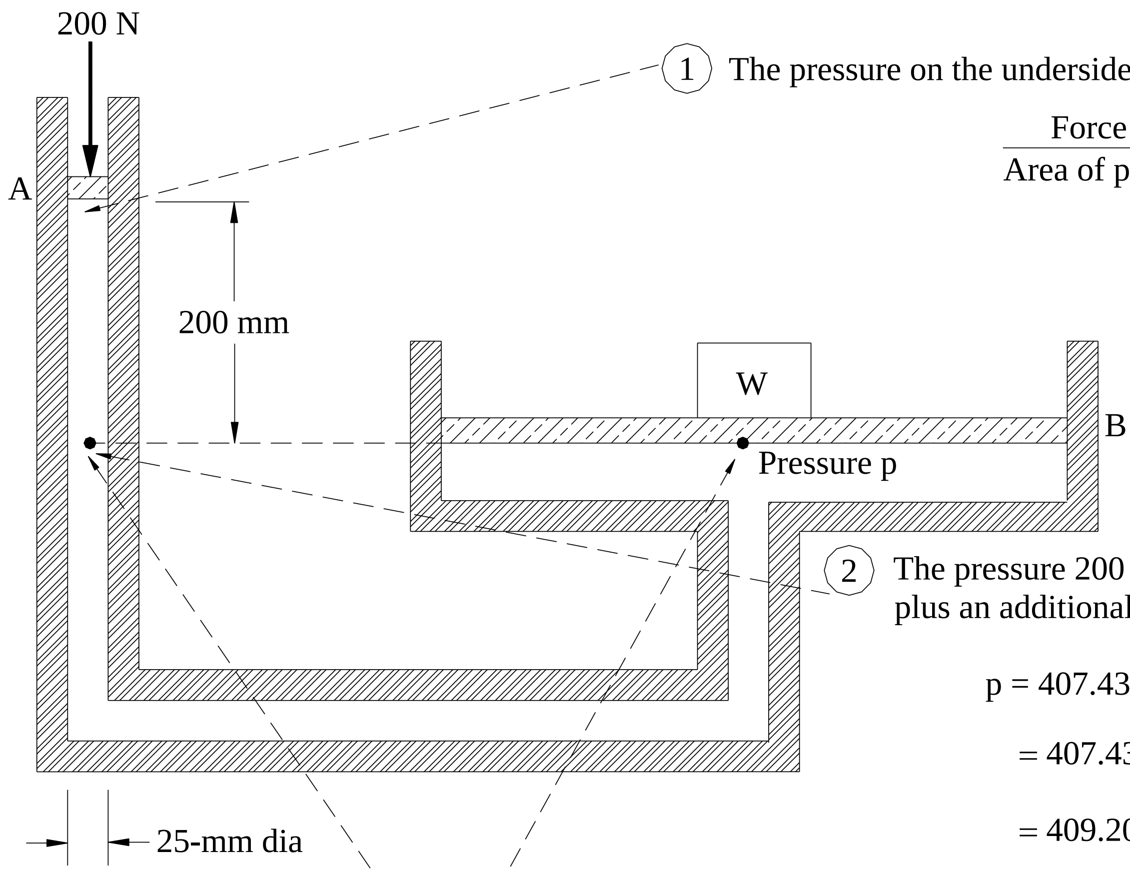
$$F_A = 294 \text{ kN} \qquad \leftarrow \text{Ans.}$$

2. Hydraulically-operated equipment is designed to transform a relatively small input force into a much larger output force. For the system shown, determine the weight W that can be supported by the piston at B when a 200-N force is applied to the piston at A.



Density of hydraulic fluid = ρ_h = 900 kg/m^3

9.4 Resultants of Hydrostatic Pressure Forces Example 2, page 2 of 3



1 The pressure on the underside of the piston at A is

$$\frac{\text{Force}}{\text{Area of piston}} = \frac{200\text{ N}}{\frac{\pi}{4} \times (0.025\text{ m})^2} = 407\,437\text{ N/m}^2$$

$$= 407.437\text{ kN/m}^2$$

$$= 407.437\text{ kPa}$$

2 The pressure 200 mm below A equals the pressure at A plus an additional amount caused by the weight of the fluid

$$p = 407.437\text{ kPa} + (900\text{ kg/m}^3)(9.81\text{ m/s}^2)\underbrace{(200\text{ mm})}_{= 0.2\text{ m}}$$

$$= 407.437\text{ kPa} + 1.766\text{ kPa}$$

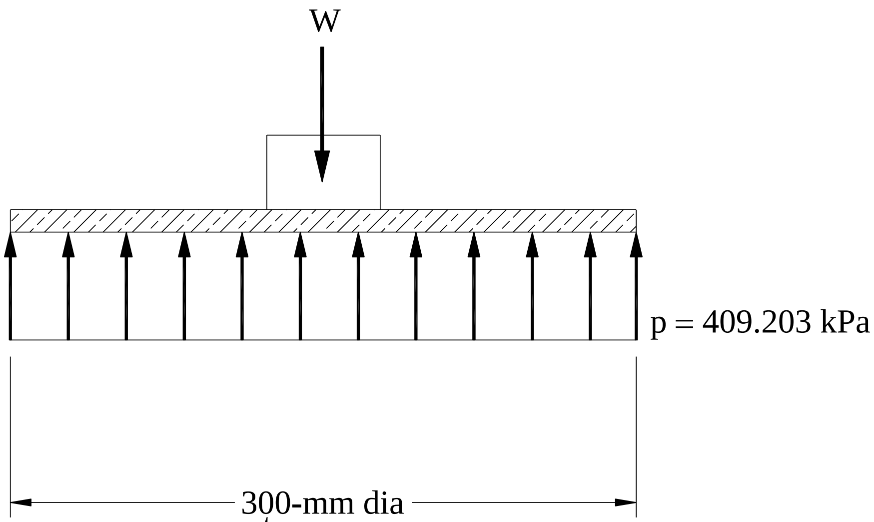
$$= 409.203\text{ kPa}$$

3 This is a typical result for hydraulic equipment. The effect of the weight of the fluid is small compared to the pressure caused by the pistons.

4 The pressure at two points at the same elevation in a fluid at rest must be equal.

9.4 Resultants of Hydrostatic Pressure Forces Example 2, page 3 of 3

5 Free-body diagram of piston B



6 Equilibrium equation

$$+\uparrow \sum F_y = -W + (409.203 \text{ kPa})[\frac{\pi}{4}(0.3 \text{ m})^2] = 0$$

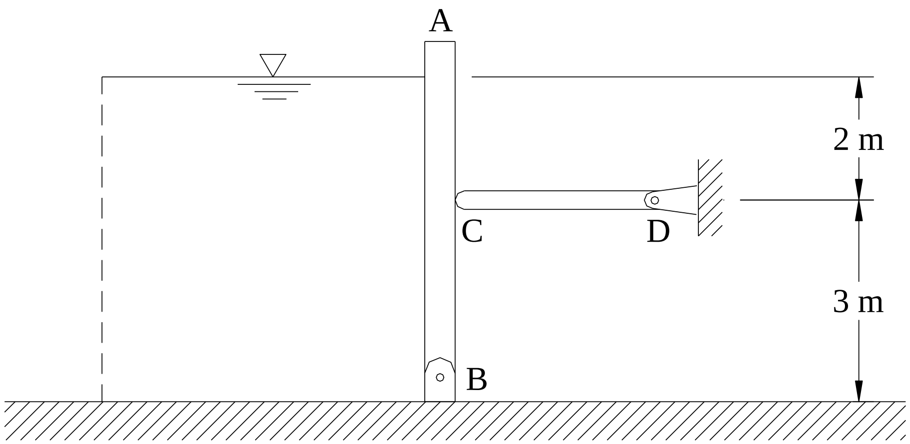
Solving gives

$W = 28.9$ kN ←Ans.

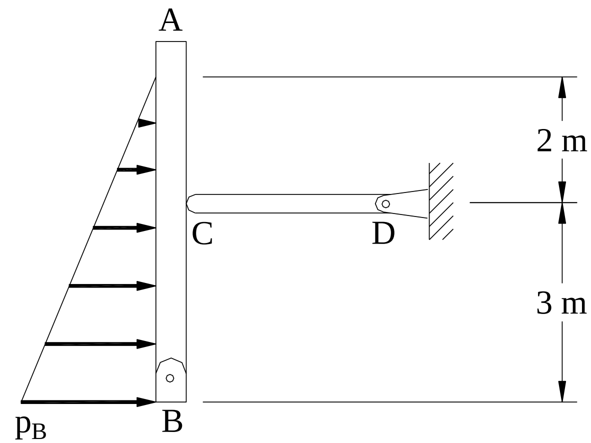
7 Thus the 200-N input force produces a 28.9 kN output force, a multiplication factor of

$$\frac{28.9 \text{ kN}}{200 \text{ N}} = \frac{28.9 \times 10^3 \text{ N}}{200 \text{ N}} = 144$$

3. During construction, gate AB is temporarily held in place by the horizontal strut CD. Determine the force in the strut, if the gate is 4-m wide.



Density of water $\rho = 10^3$ kg/m^3

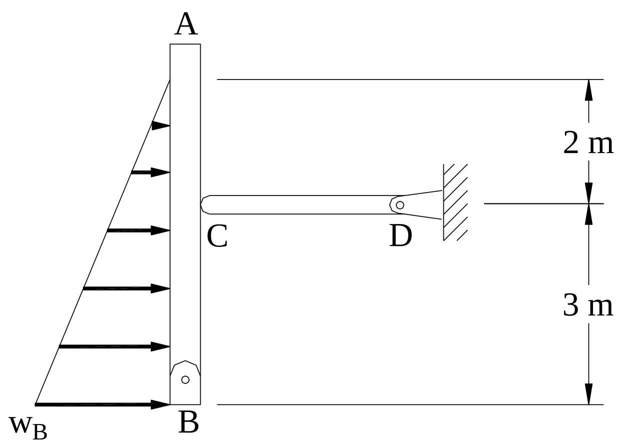


1. The water pressure varies linearly from 0 to p_B, where

$$p_B = \rho g \times (2\ \text{m} + 3\ \text{m})$$

$$= (10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})$$

$$= 49.05\ \text{kPa}$$



2. Convert the pressure to a force per length by multiplying by the width, 4 m:

$$w_B = p_B \times 4\ \text{m}$$

$$= (49.05\ \text{kPa}) \times 4\ \text{m}$$

$$= 196.20\ \text{kN/m} \qquad (1)$$

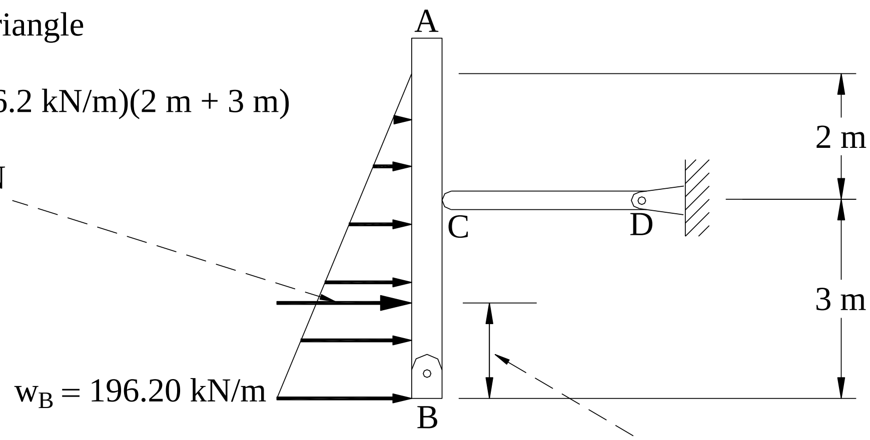
3 Resultant of distributed load

= area under load curve

= area of triangle

= (1/2)(196.2 kN/m)(2 m + 3 m)

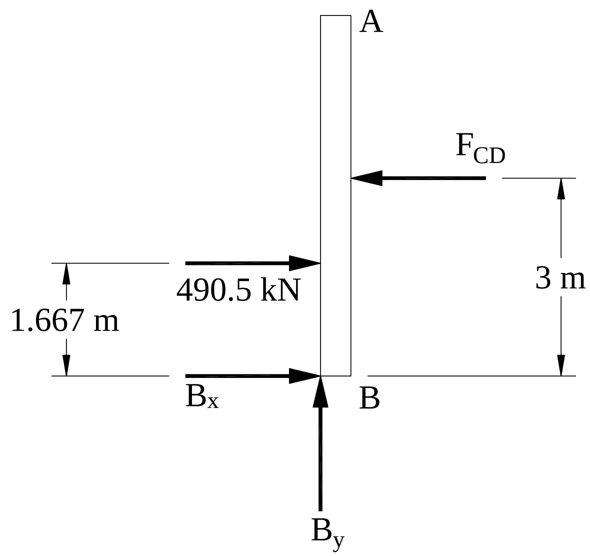
= 490.5 kN



4 The resultant passes through the centroid of the triangle, or one-third of the distance from the base to the opposite vertex:

(1/3)(2 m + 3 m) = 1.667 m above point B

5 Free-body diagram of gate AB



6 Equilibrium equation

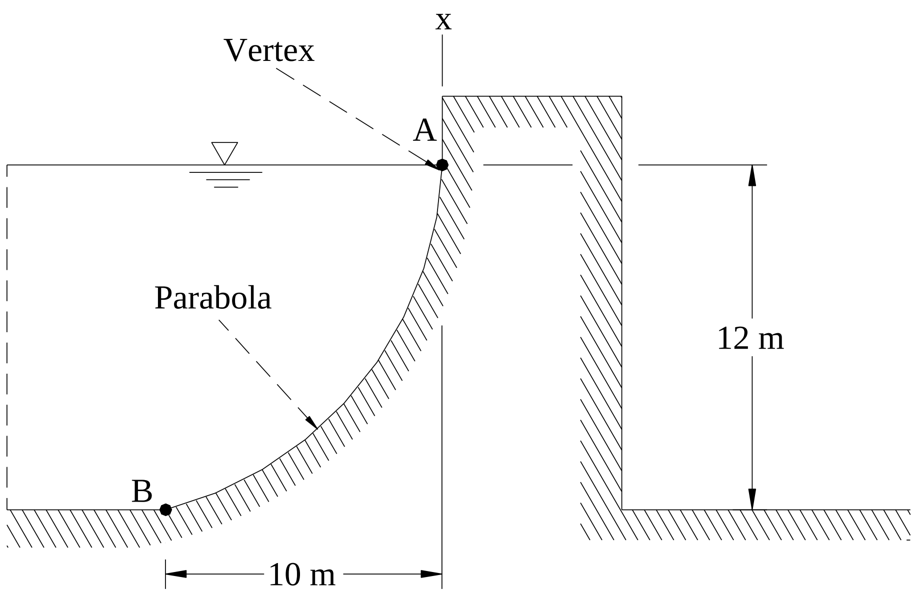
$$\circlearrowleft + \Sigma M_B = (F_{CD})(3\text{ m}) - (490.5\text{ kN})(1.667\text{ m}) = 0$$

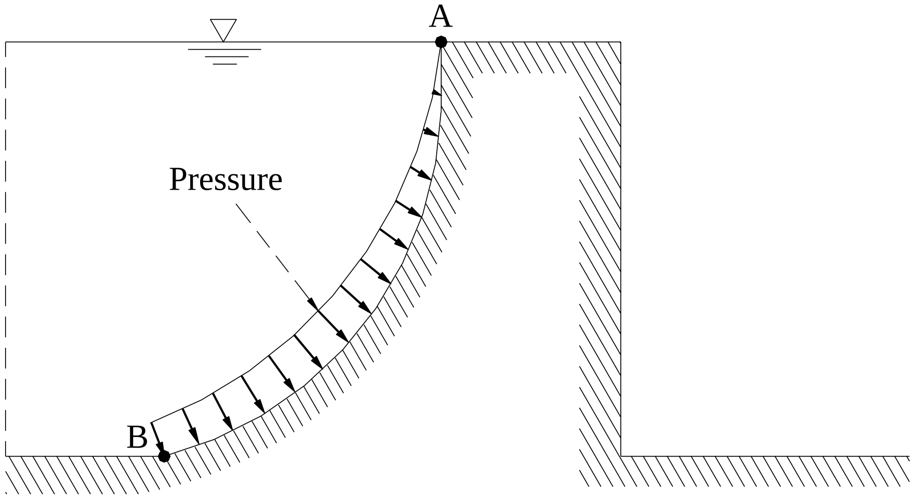
solving giving

$$F_{CD} = 273\text{ kN} \qquad \leftarrow\text{Ans.}$$

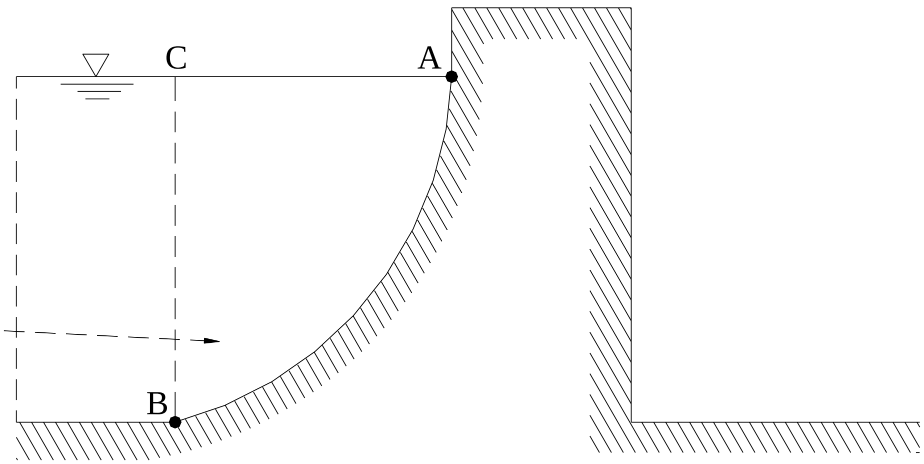
This is a large force. Most likely more than one strut would be used.

4. Determine the magnitude and line of action of the resultant hydrostatic force acting on a 1-m wide section of the seawall. Assume that the density of sea water is $\rho = 1.02 \times 10^3$ kg/m^3.





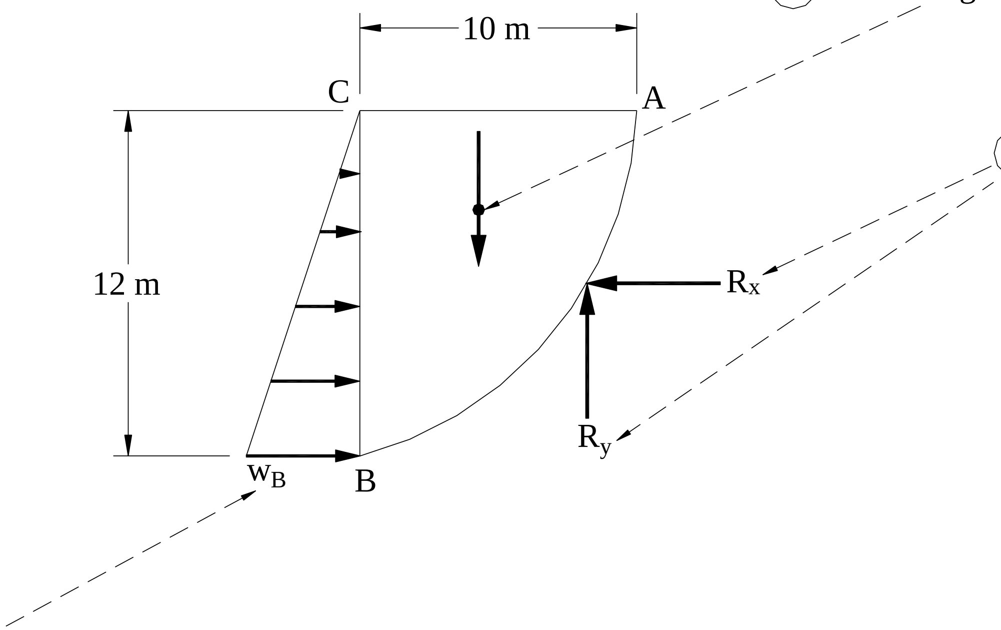
1 Integration can be used to calculate the resultant of the pressure forces acting on the curved wall, but it is easier to consider a free body consisting of a portion of the water behind the sea wall.



2 Free-body diagram of region of ABC

4 W = weight of water

5 Components of force *from* seawall *acting on* water



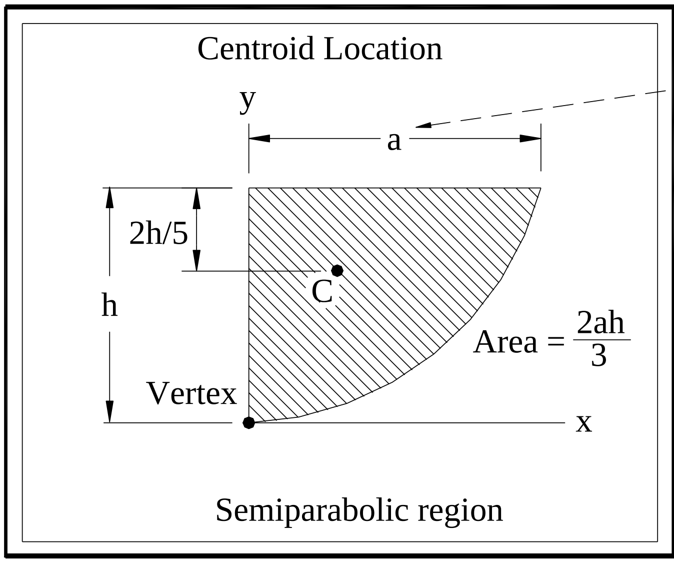
3 Force per length, w_B = (pressure at B) × (1-m distance normal to plane of figure)

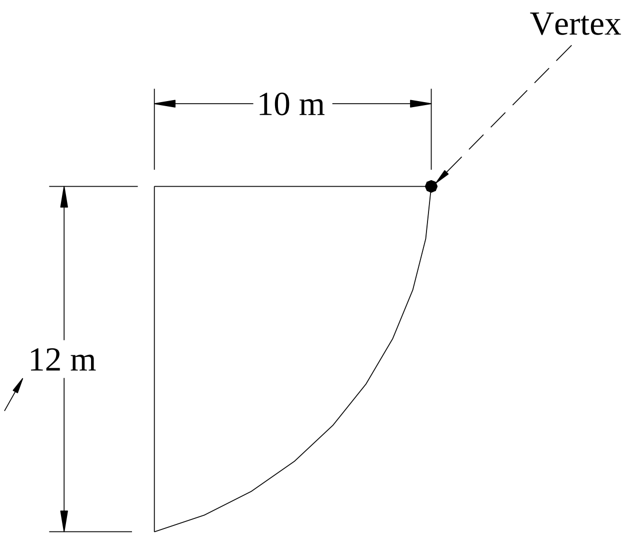
$$= (\rho g \times 12 \text{ m})(1 \text{ m})$$

$$= (1.02 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(12 \text{ m})(1 \text{ m})$$

$$= 120.1 \text{ kN/m}$$

6 The magnitude of the weight, W, can be calculated by using information from a table of properties of planar regions as shown below:





7 In our particular example, the length of the side opposite the vertex = 12 m = a, and h = 10 m, so

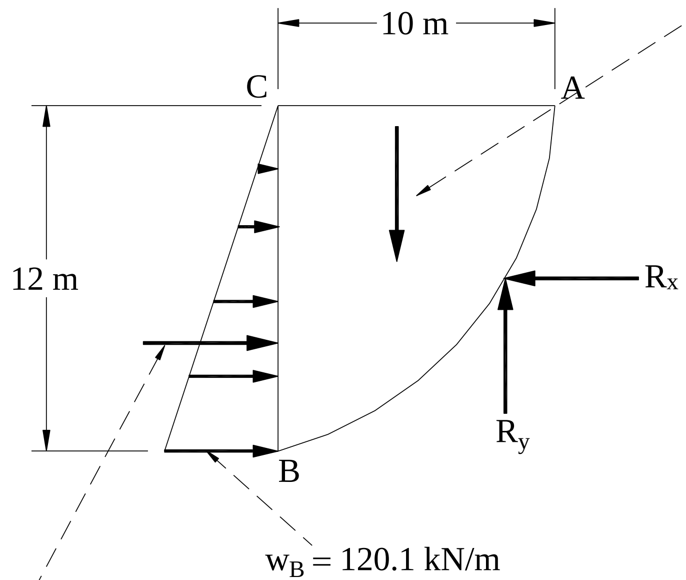
$$\text{Area} = 2ah/3$$

$$= 2(12\text{ m})(10\text{ m})/3$$

$$= 80\text{ m}^2 \qquad (1)$$

9.4 Resultants of Hydrostatic Pressure Forces Example 4, page 5 of 10

8 Free-body diagram of region ABC (repeated)



9 P = resultant of pressure forces

= area of w-diagram

= (1/2)(12 m)(120.1 kN/m)

= 720.6 kN

10 W = weight

$= \rho \times g \times \text{area} \times \text{width}$

$= (1.02 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(80 \text{ m}^2)(1 \text{ m})$

= 800.5 kN

11 Equilibrium equations

$\xrightarrow{+} \sum F_x = 0$: $-R_x + 720.6 \text{ kN} = 0$ (2)

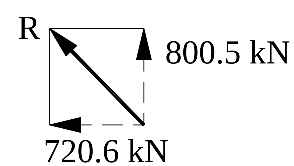
$+\uparrow \sum F_y = 0$: $R_y - 800.5 \text{ kN} = 0$ (3)

Solving gives

$R_x = 720.6$ kN

$R_y = 800.5$ kN

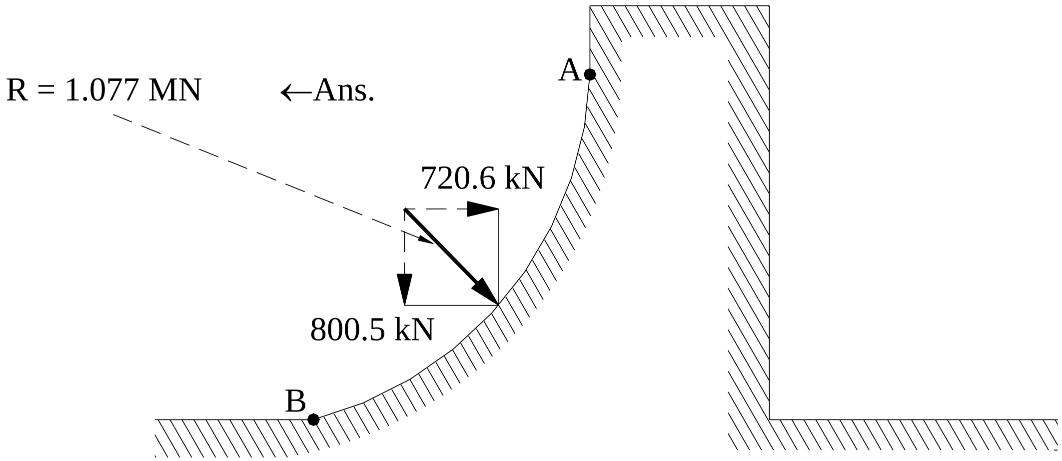
12 Resultant



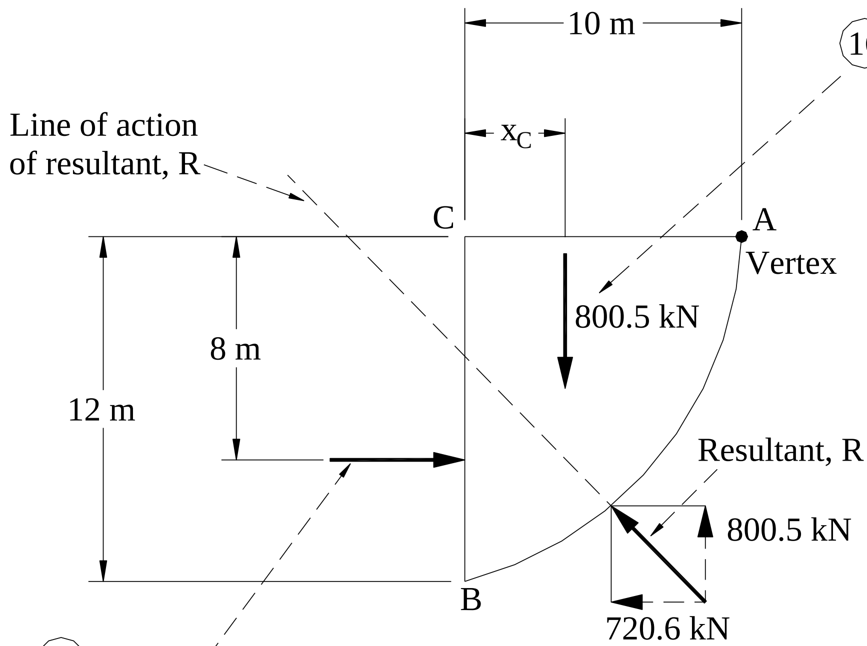
$$R = \sqrt{(720.6 \text{ kN})^2 + (800.5 \text{ kN})^2}$$

= 1.077 MN

(13) R is the force of the seawall on the water. The force of the water on the seawall is equal and opposite.



14 To determine the line of action of the resultant force, consider a free-body diagram of region ABC again.

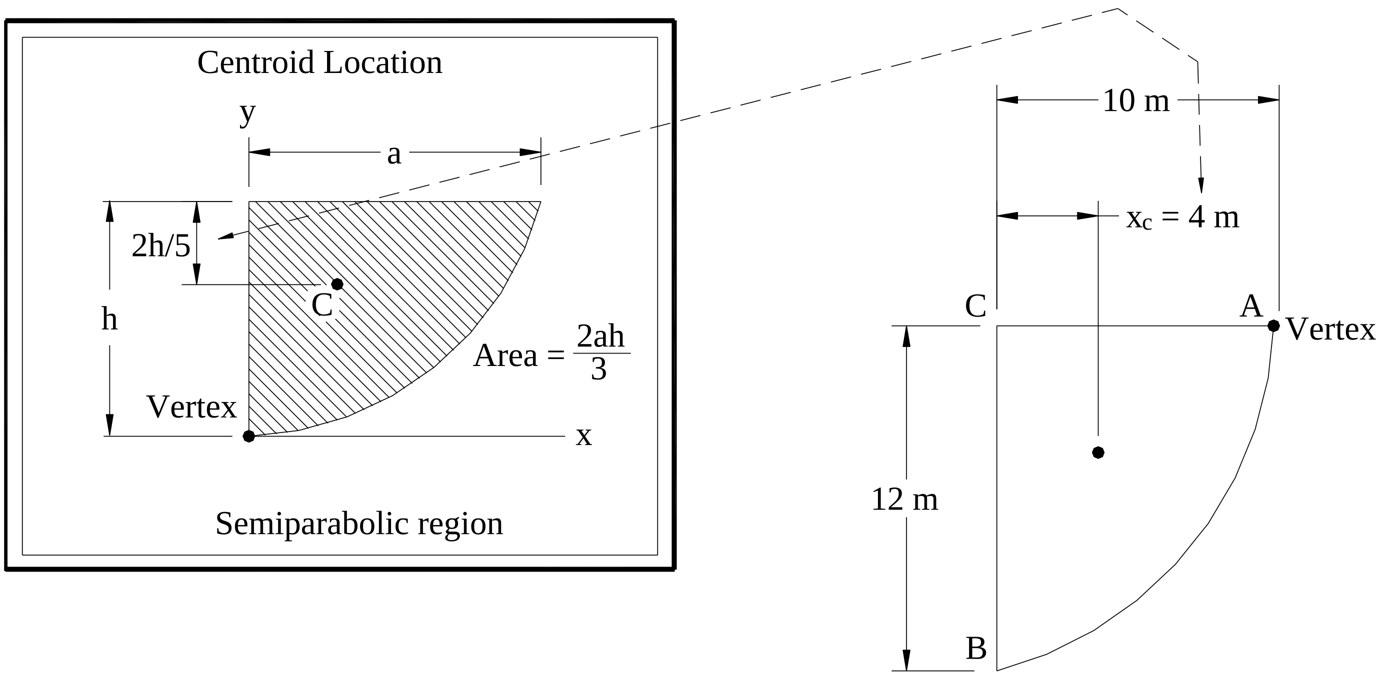


16 The 800.5 kN weight of the water acts through the centroid of the semiparabolic area. The centroidal distance x_c can be calculated using a table giving geometric properties of common areas.

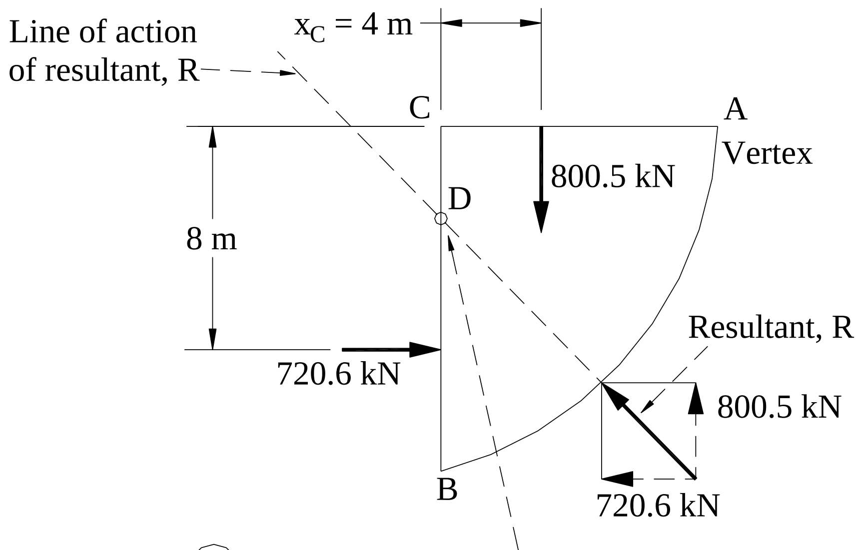
15 Resultant of triangular distribution of pressure equals 720.6 kN and acts 8 m [= (2/3)(12 m)] from point C.

17 To locate the centroid, we again make use of a table of properties of planar regions as shown below:

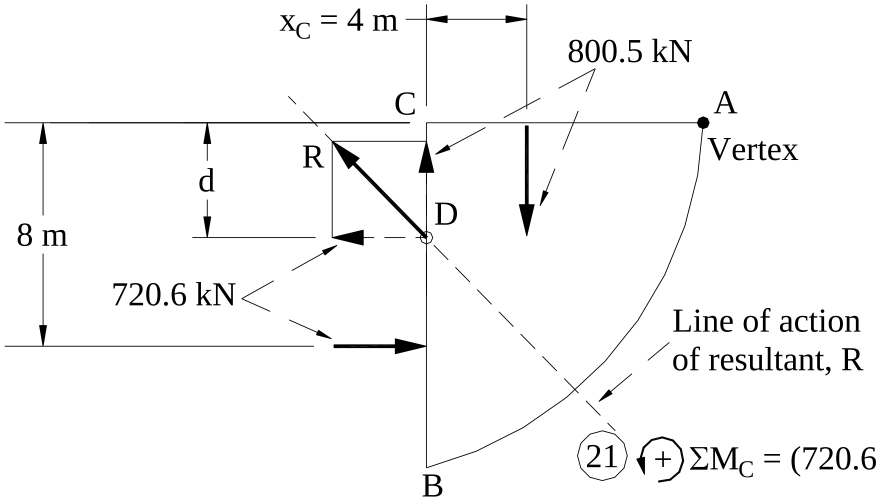
18 In our particular example, the length of the straight side *adjacent* to the vertex = 10 m = h, and so the centroid is located 4 m [= (2/5)(10)] to the right of of C.



19 The free-body diagram of region ABC can be re-drawn with the known distances specified.



20 A force can be considered to act anywhere along its line of action (Principle of transmissibility). We can simplify our calculations by considering the resultant R to act at point D where the line of action intersects the left edge of the free body.

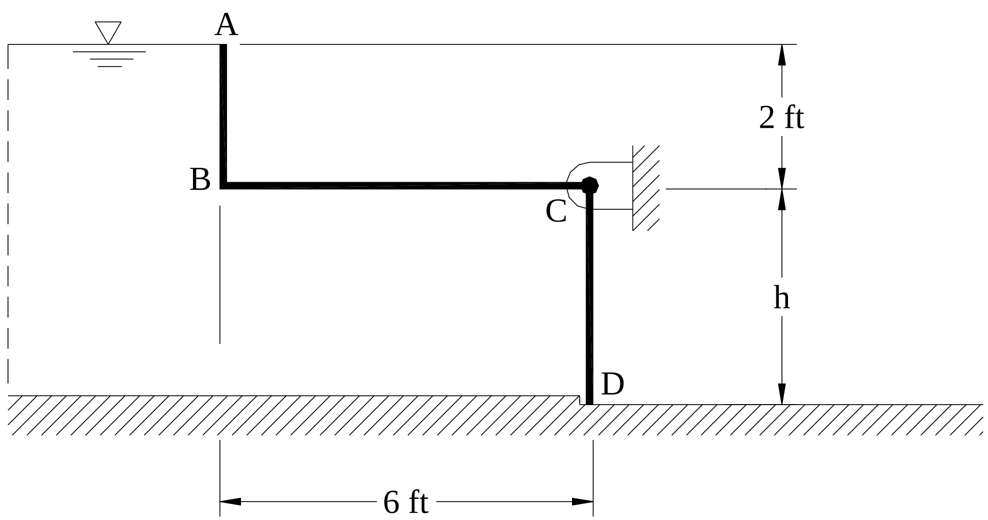


21 $\circlearrowleft + \Sigma M_C = (720.6\text{ kN})(8\text{ m}) - (720.6\text{ kN})(d) - (800.5\text{ kN})(4\text{ m}) = 0$

Solving gives the vertical distance below point C to the line of action of the resultant:

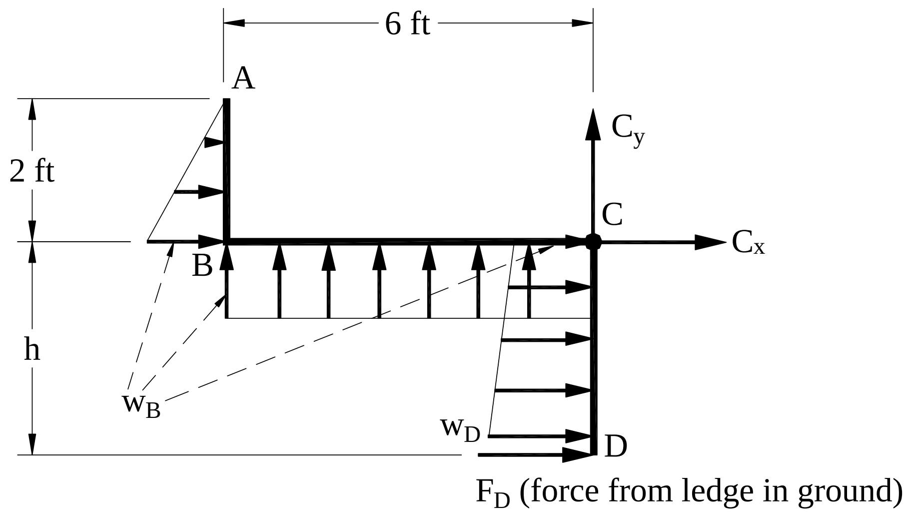
$d = 3.56$ m $\qquad\leftarrow$Ans.

5. Determine the distance h for which the gate is just about to open. Neglect the weight of the gate. The specific weight of the fluid is γ.



1 The value of h will not depend on the gate width (distance measured perpendicular to the plane of the figure) because the width would cancel out of the equation for the sum of moments about the support C. Accordingly we will base our calculations on a 1-ft width of gate.

2 Free-body diagram of gate (distributed forces)



3 Pressure:

$p_B = (2 \text{ ft}) \times \gamma$

$p_D = (2 \text{ ft} + h) \times \gamma$

1-ft width

4 Force per length:

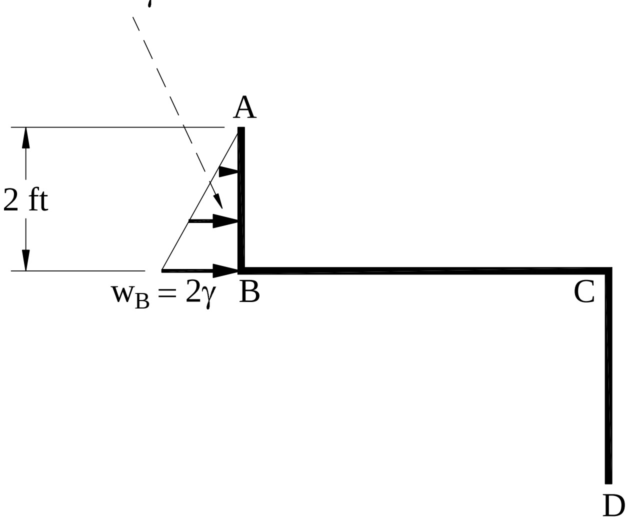
$w_B = p_B \times (1 \text{ ft}) = (2\gamma) \times 1 = 2\gamma$

$w_D = p_D \times (1 \text{ ft}) = (2 + h)\gamma \times 1 = (2 + h)\gamma$

5 Resultant force on AB

Area = (1/2)(2 ft)(2γ)

= 2γ



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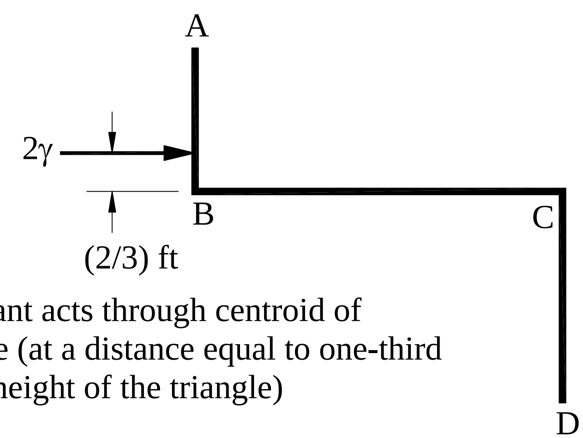
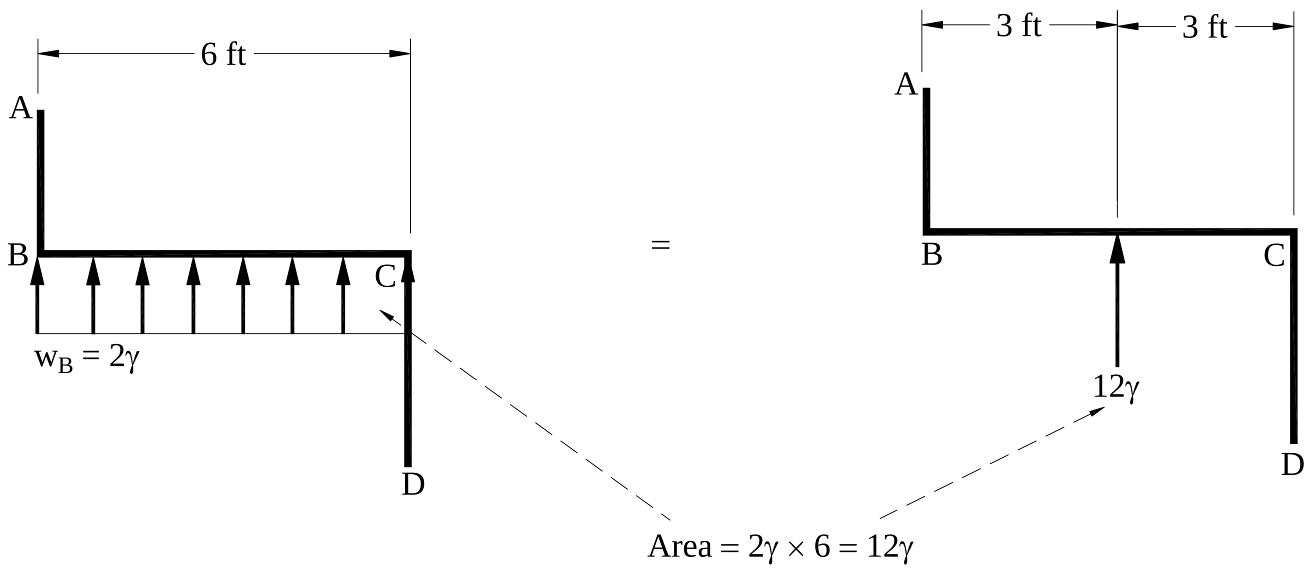
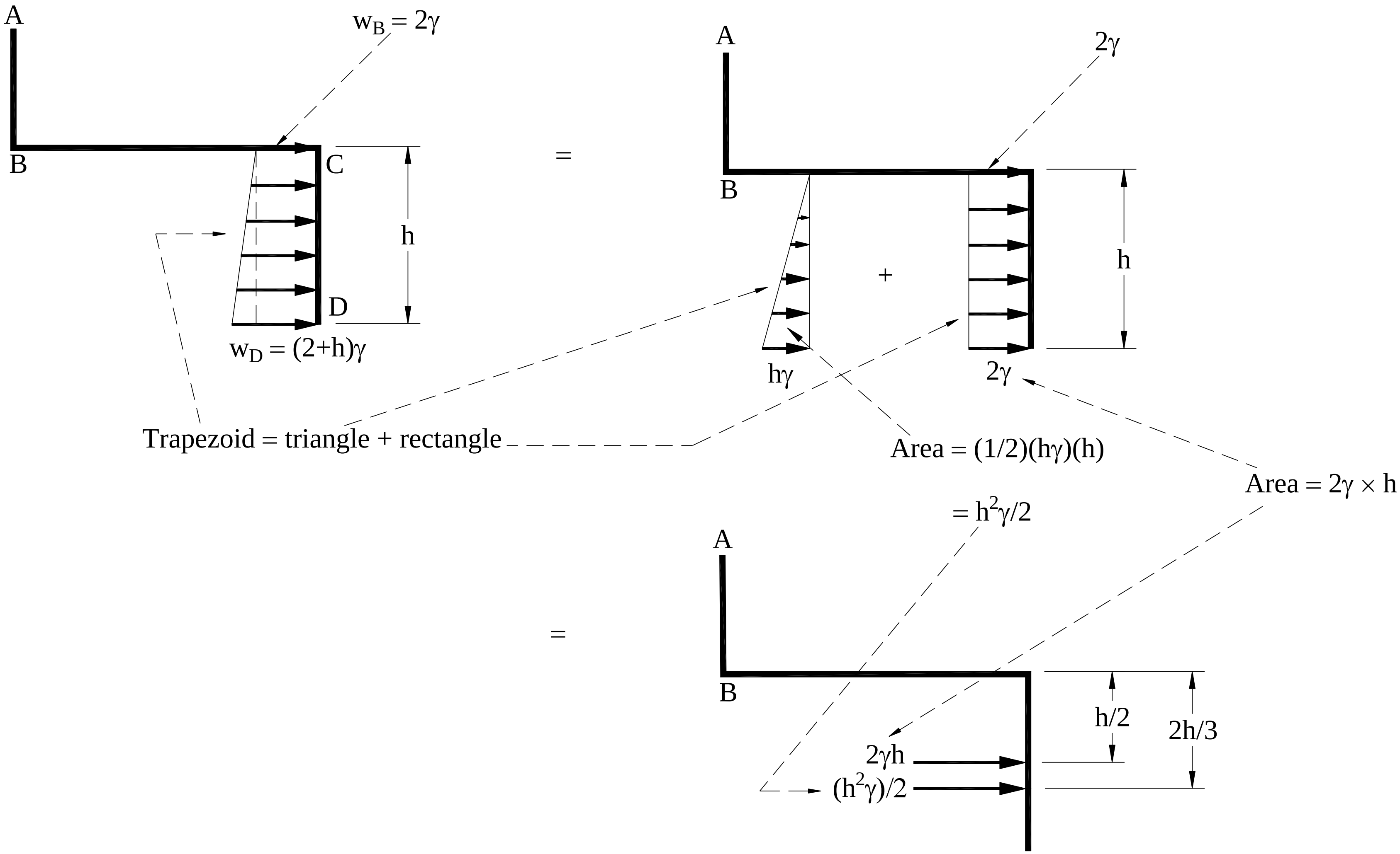


6 Resultant acts through centroid of triangle (at a distance equal to one-third of the height of the triangle)

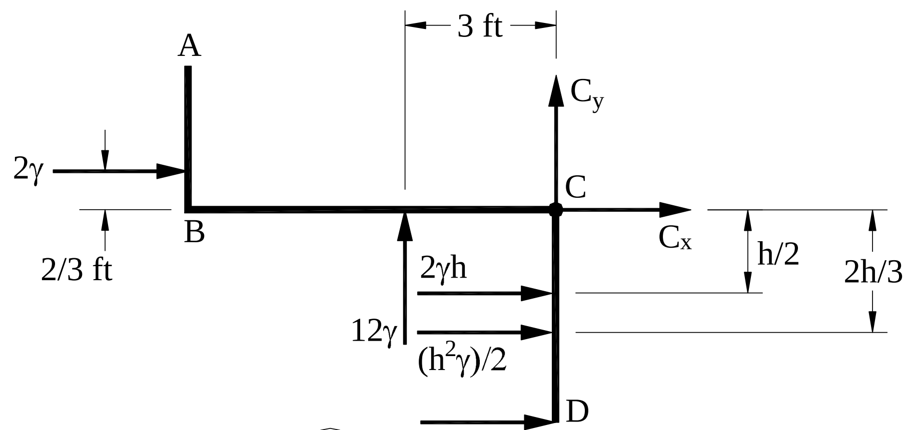
7 Resultant force on BC



8 Resultant force on CD



9 Free-body diagram (resultant forces)



10 $F_D = 0$, since gate is just about to open

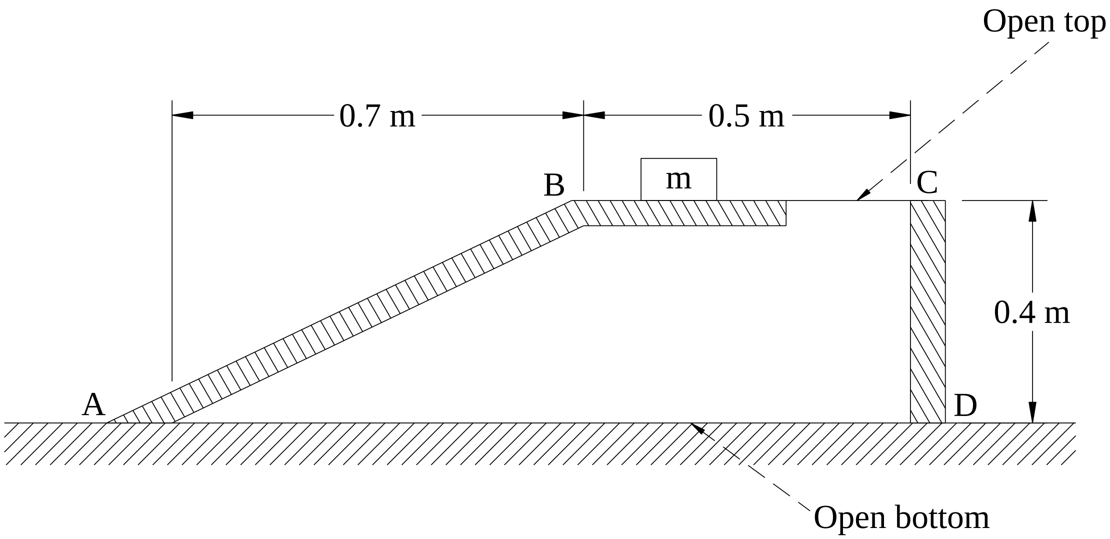
11 Equilibrium equation

$$\circlearrowleft + \Sigma M_C = 0: \; (-2\gamma)(2/3 \text{ ft}) - (12\gamma)(3 \text{ ft}) + (2\gamma h)(h/2) + (h^2\gamma/2)(2h/3) = 0$$

Solving gives

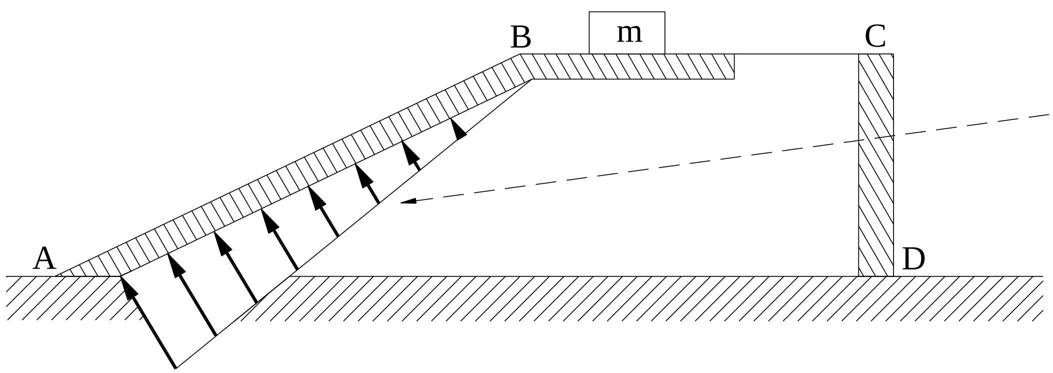
$h = 4.0$ ft ←Ans.

6. Concrete is poured into the open top of a form to produce a ramp. If the ramp is 0.8-m wide, determine the minimum mass m needed to keep the form from lifting off the ground.

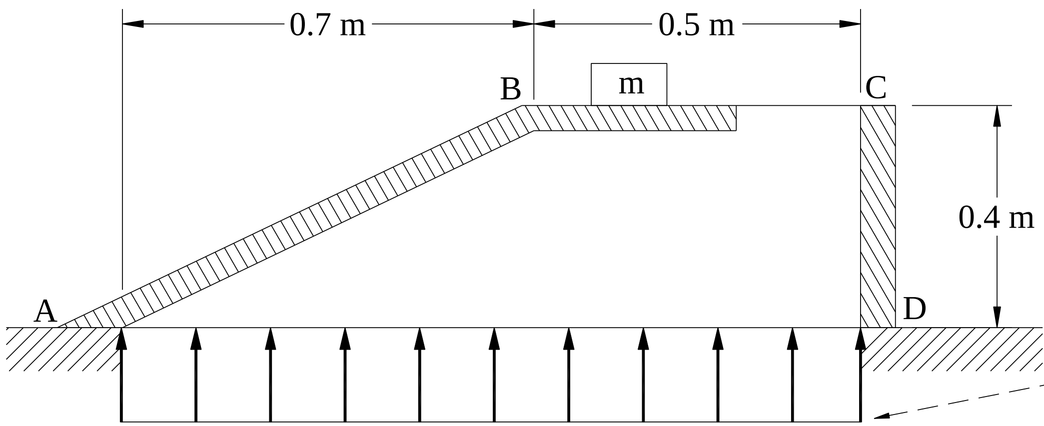


Mass of form = 60 kg

Density of concrete = $\rho_c = 2.4 \times 10^3$ kg/m^3



1 We can solve the problem by finding the resultant of the pressure of the concrete acting on part AB of the form. But we would have to compute the length AB and also compute the angle the resultant makes with the vertical. Although the problem can be done in this manner, it is easier to consider a free body consisting of the form together with the concrete. Thus let's first find the resultant force of the pressure of the floor pushing up on the concrete.



2 Pressure at D $= (2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$
$\times (0.4 \text{ m})$

$$= 9.418 \times 10^3 \text{ N/m}^2$$

Distributed load (force per length) =

pressure $\times$ width of gate

$$= (9.418 \times 10^3 \text{N})(0.8 \text{ m})$$

$$= 7.534 \times 10^3 \text{ N/m}$$

Resultant force $= (7.534 \times 10^3 \text{ N/m})$
$\times (0.7 \text{ m} + 0.5 \text{ m})$

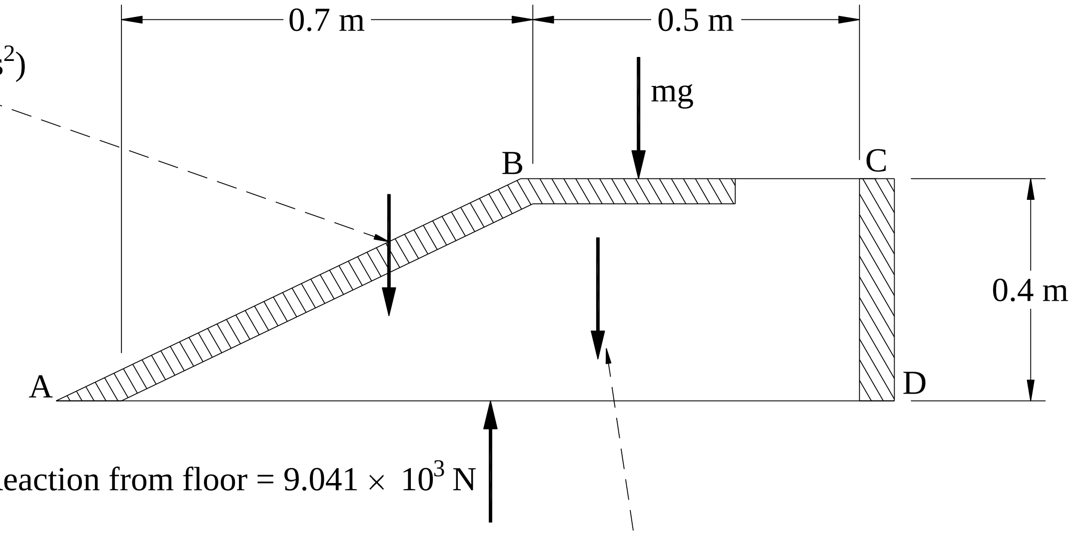
$$= 9.041 \times 10^3 \text{ N}$$

9.4 Resultants of Hydrostatic Pressure Forces: Example 6, page 3 of 3

(3) Free-body diagram of form together with concrete

(4) Weight of form = (60 kg)(9.81 m/s^2)

= 588.6 N



(5) Weight of concrete = $\rho_c g$ × volume

$$= (2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)[(1/2)(0.7 \text{ m}) \times (0.4 \text{ m})(0.8 \text{ m}) + (0.5 \text{ m})(0.4 \text{ m})(0.8 \text{ m})]$$

$$= 6.404 \times 10^3 \text{ N}$$

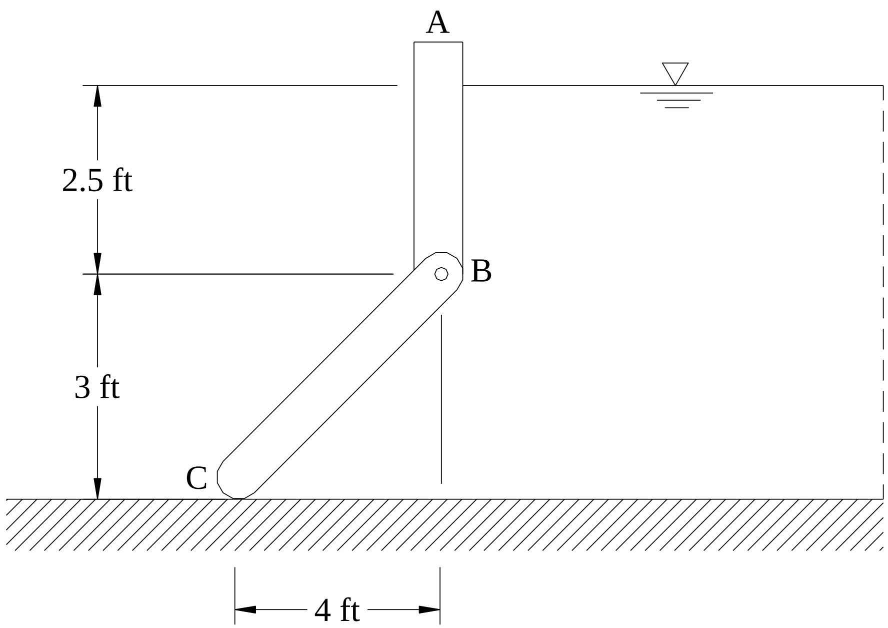
(6) Equilibrium equation

$$+\uparrow \sum F_y = 0: \; 9.041 \times 10^3 \text{ N} - mg - 588.6 \text{ N} - 6.404 \times 10^3 \text{ N} = 0$$

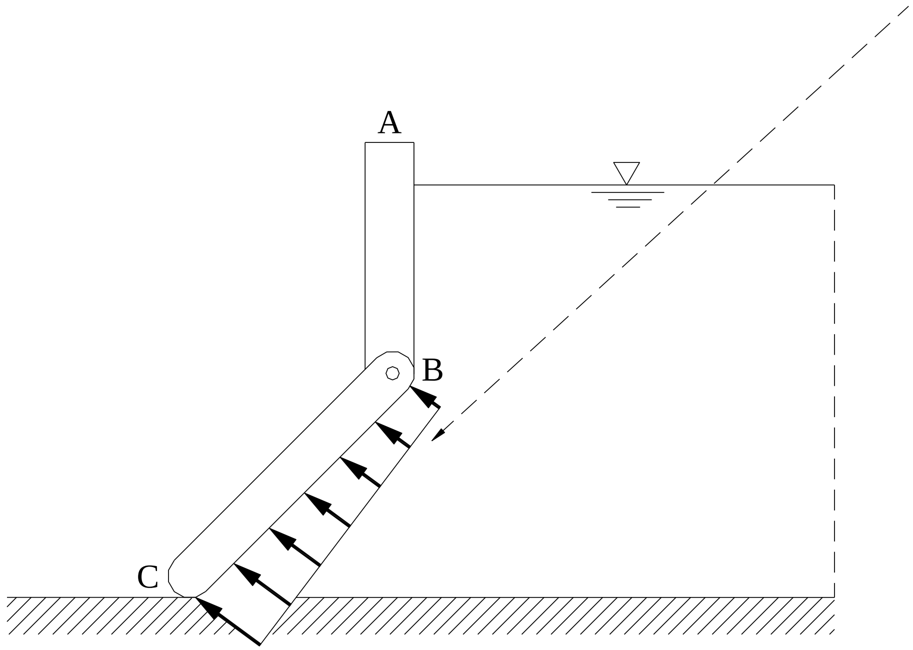
Setting g equal to 9.81 m/s^2 and solving gives

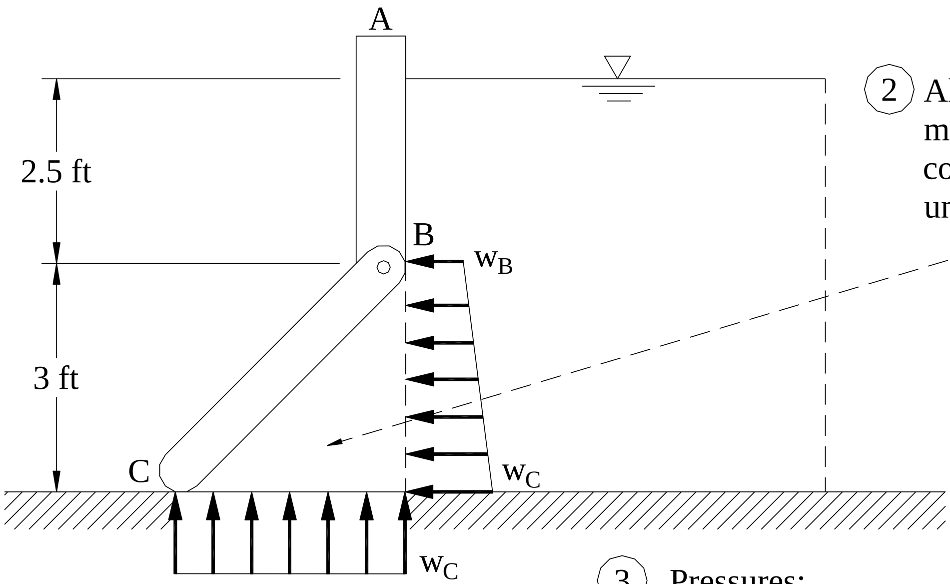
m = 209 kg ←Ans.

7. Determine the minimum weight W of gate BC required to keep the gate closed. The gate is 2-ft wide and of uniform density. The specific weight of water is $\gamma = 62.4\ \text{lb/ft}^3$.



1 If we attempt to solve the problem by considering the pressure acting directly on BC, then we have to calculate the length and angle of inclination of BC.





2 Although the problem can be solved in this manner, it is easier to consider a free body consisting of gate BC *together with* the water under BC.

3 Pressures:

at B,

$$p_B = \gamma \times 2.5 \text{ ft} = 2.5\gamma$$

at C,

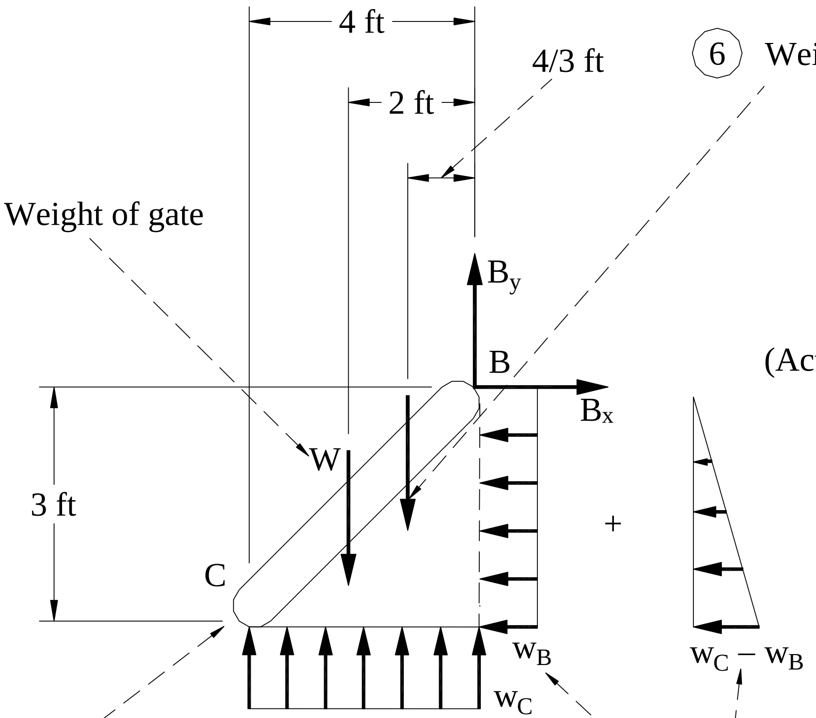
$$p_C = \gamma \times (2.5 \text{ ft} + 3 \text{ ft}) = 5.5\gamma$$

Distributed loads (force per length):

$$w_B = p_B \times (\text{width of gate}) = (2.5\gamma)(2 \text{ ft}) = 5\gamma \qquad (1)$$

$$w_C = p_C \times (\text{width of gate}) = (5.5\gamma)(2 \text{ ft}) = 11\gamma \qquad (2)$$

4 Free-body diagram of gate BC together with the water under BC



5 Weight of gate

6 Weight of water

$= \gamma \times$ Volume

$= \gamma(1/2)(4\text{ ft})(3\text{ ft})(2\text{ ft})$

$= 12\gamma$

(Acts through centroid of triangle)

7 The force from the ground at C acting on the gate is zero since the gate is about to lose contact with ground.

8 The trapezoidal distributed load from the fluid acting on the right side of the free body can be considered to be the sum of a rectangular and triangular distributed load.

9 Equation of equilibrium

$$\circlearrowleft + \Sigma M_B = 0: \; W(2 \text{ ft}) + 12\gamma(4/3 \text{ ft}) - 4w_C(2 \text{ ft}) - 3w_B(3/2 \text{ ft}) - (1/2)(w_C - w_B)(3 \text{ ft})(2/3 \times 3 \text{ ft}) = 0 \qquad (3)$$

Resultant force from uniform load on bottom

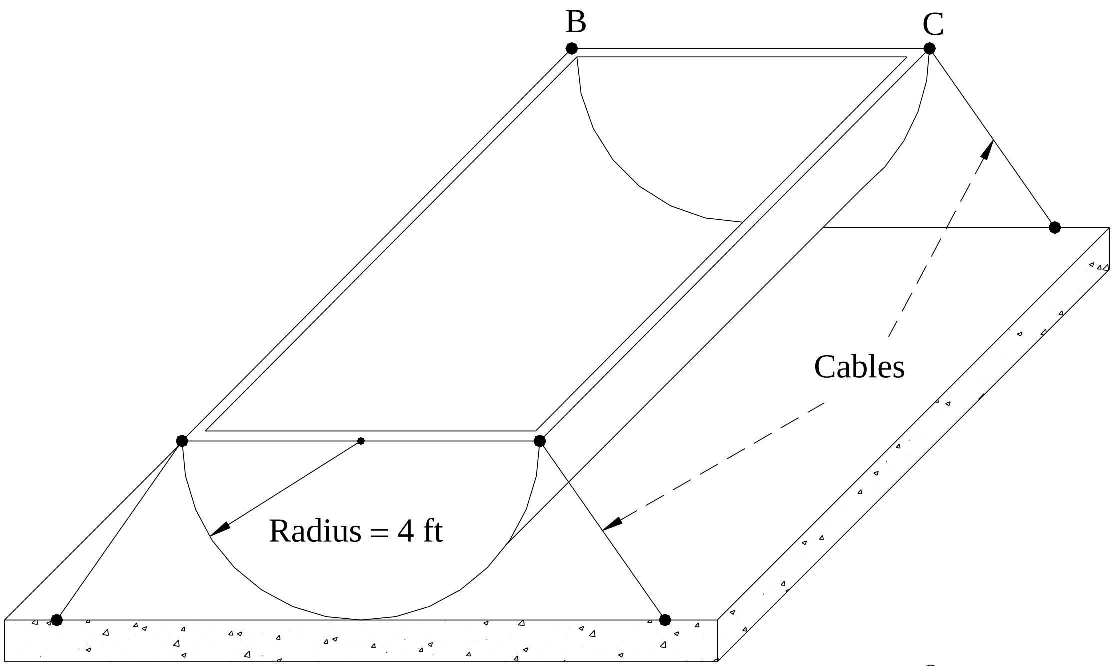
Resultant force from uniform load on right side

Resultant force from triangular load on right side

10 Substituting $w_B = 5\gamma$ (Eq. 1), $w_C = 11\gamma$ (Eq. 2), and $\gamma = 62.4$ lb/ft^3 into Eq. 3 and solving gives

$W = 3{,}510$ lb ←Ans.

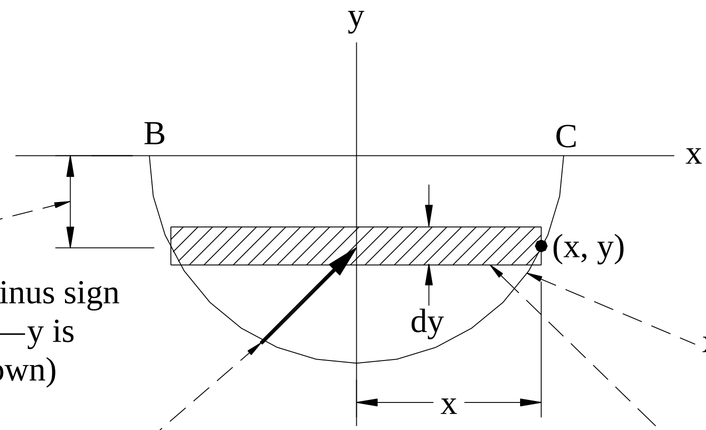
8. Determine the magnitude and line of action of the resultant hydrostatic force acting on the semicircular end of the tank. The tank is filled to the top with water. The specific weight of water is $\gamma = 62.4$ lb/ft^3.



1 Because the end of the tank is not of uniform width, we have to use integration to compute the magnitude of the resultant.

9.4 Resultants of Hydrostatic Pressure Forces: Example 8, page 2 of 4

(2) Pressure force on elemental area dA



(5) Distance = $-y$ (Insert a minus sign to get a positive distance—y is negative in the region shown)

$x^2 + y^2 = 4^2$

(4) Solve for x and substitute

(3) $dA = 2x\,dy$

$= 2\sqrt{4^2 - y^2}\,dy$

(6) Differential force:

$dR = \text{pressure} \times \text{area}$

$= [\gamma(-y)] \times [2\sqrt{4^2 - y^2}\,dy]$

(7) Resultant force:

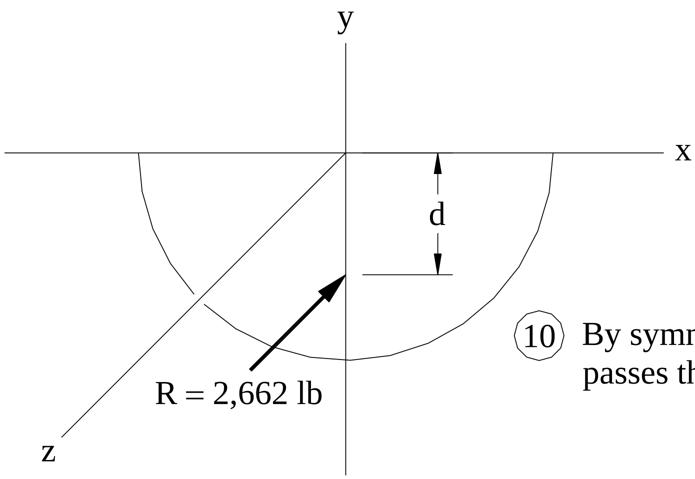
$R = \int dR$

$= \int_{-4}^{0} [\gamma(-y)]\,[2\sqrt{4^2 - y^2}\,dy]$

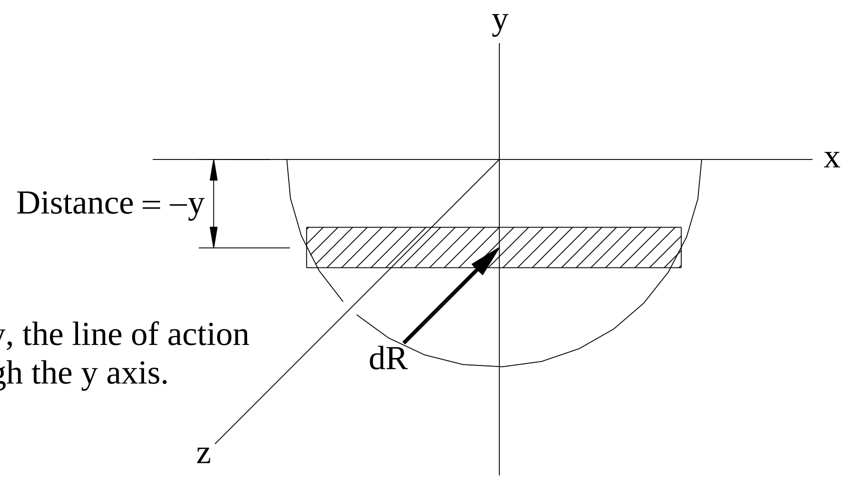
$= 2{,}662$ lb ←Ans.

(8) Use $\gamma = 62.4$ lb/ft^3 and evaluate with the integral function on a calculator.

9 To determine the line of action of the resultant, equate the moment of the resultant to the integral of the moment of the differential force, dR.



10 By symmetry, the line of action passes through the y axis.



11 Moment of R about x axis = Integral of moment of dR about x axis

$$R \times d = \int (-y) \times dR \qquad (1)$$

or

$$2{,}662 \text{ lb} \times d = \int_{-4}^{0} (-y)\,[-2\underbrace{\gamma}_{62.4 \text{ lb}} y\sqrt{4^2 - y^2}\,]\,dy$$

Evaluating the integral by using the integral function on a calculator and then solving for d gives

$$d = 2.36 \text{ ft} \qquad \leftarrow \text{Ans.}$$

12 We could have saved some work by using a table of moments of inertia as follows: Eq. 1 is

$R \times d = \int (-\,y) \times dR$ (Eq.1 repeated)

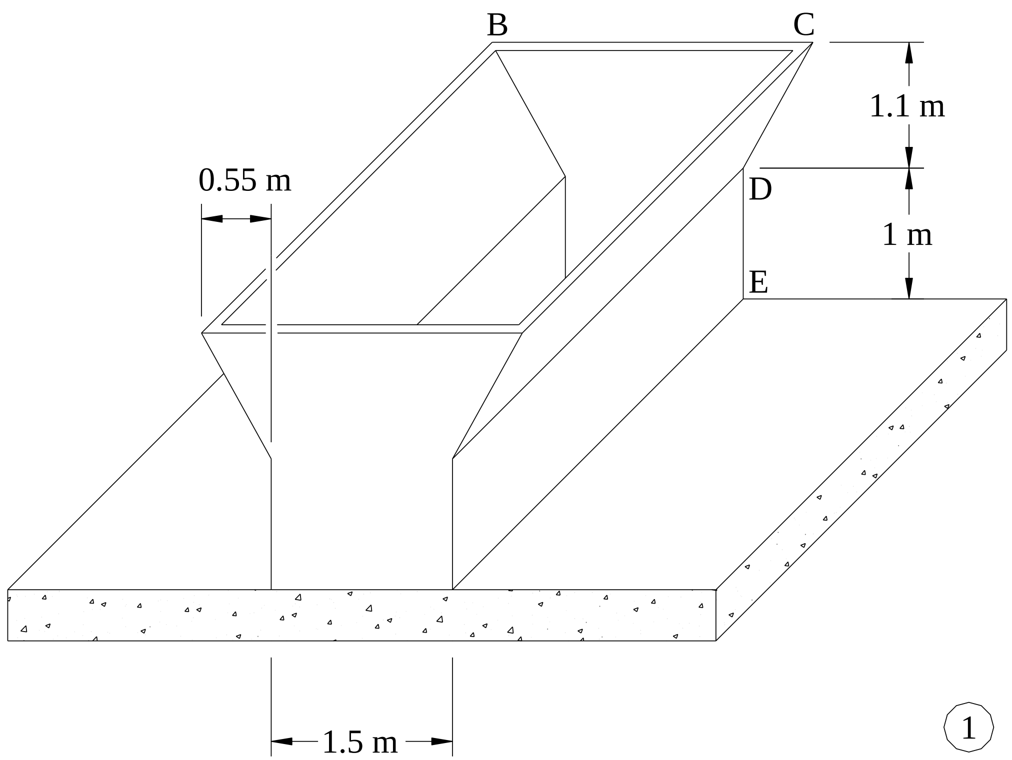
$= \text{pressure} \times dA$

$= [\gamma(-y)]\, dA$

$= \gamma \underbrace{\int y^2 dA}$

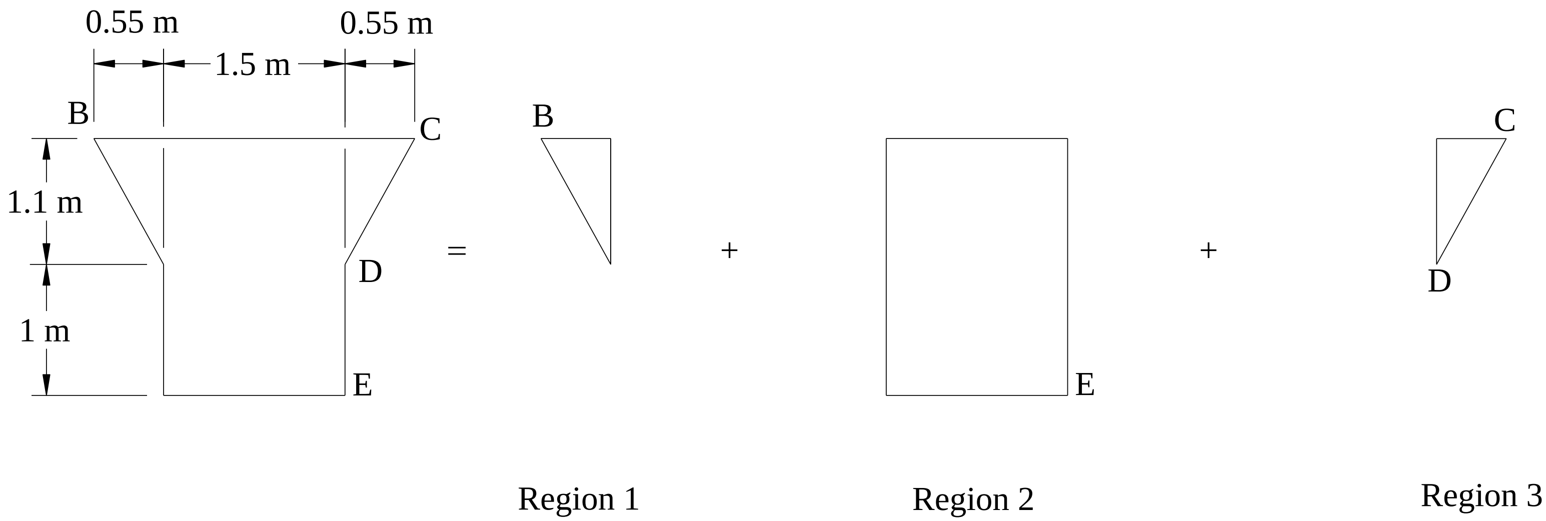
$= I_x$, the moment of inertia of the area about the x axis.

9. Determine the magnitude and line of action of the resultant hydrostatic force acting on the end of the tank, which is filled to the top with water. The density of water is $\rho = 10^3$ kg/m^3.



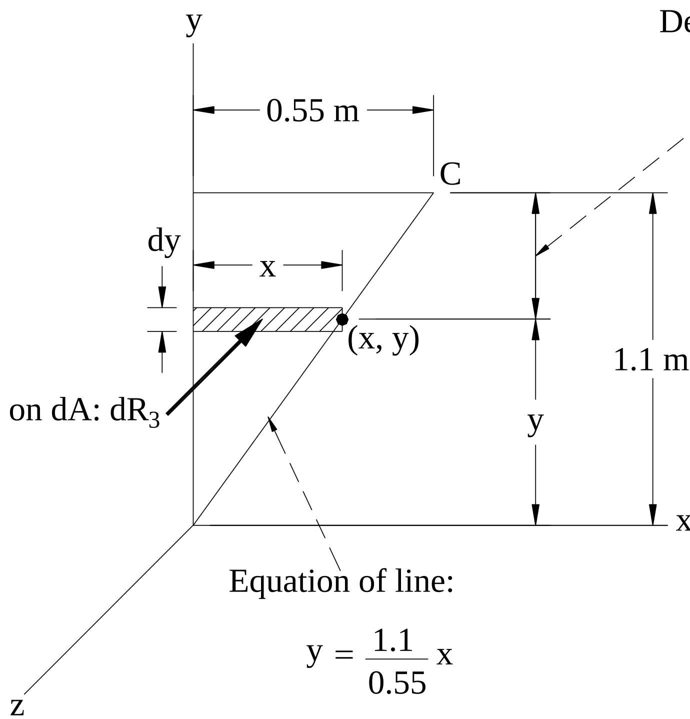
1 Because the end of the tank is not of uniform width, we have to use integration to compute the magnitude of the resultant.

2 Let's divide the end of the tank into three regions.



3 Because of symmetry, Regions 1 and 3 have the same resultant force. Thus we need to consider only one of the regions. Let's choose Region 3.

4 Calculation of resultant of pressure on Region 3



Pressure force acting on dA: dR_3

Depth below free surface

$= 1.1\text{ m} - y$

Equation of line:

$$y = \frac{1.1}{0.55}x$$

$$= 2x$$

5 Elemental area:

$$dA = x \times dy$$

$$= (y/2)\,dy$$

Solve for x and substitute in the expression for dA.

6 Differential force

$$dR_3 = \text{pressure} \times \text{area}$$

$$= [\rho g(1.1 - y)] \times [(y/2)\,dy] \qquad (1)$$

7 Resultant force for Region 3

$$R_3 = \int dR_3$$

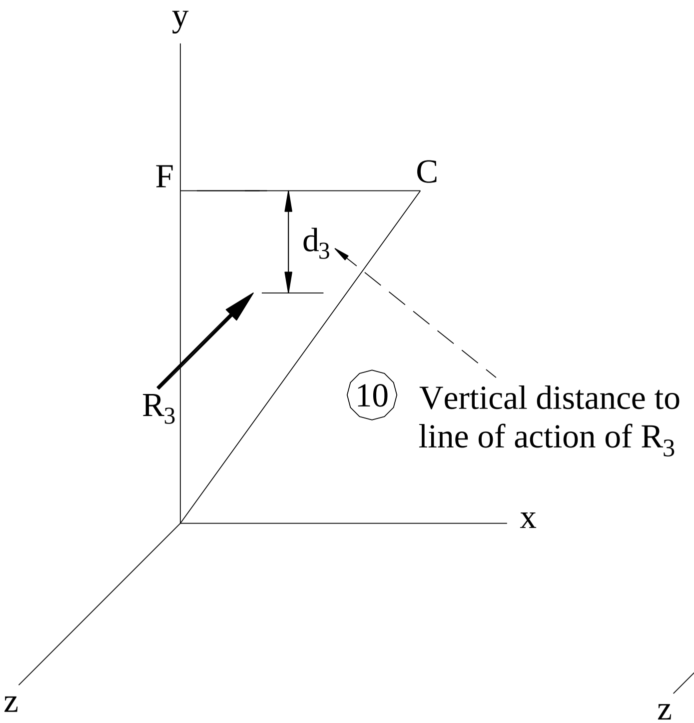
$$= \rho g \int_0^{1.1} \overbrace{(1.1 - y)\,(y/2)\,dy}^{\text{Eq. 1}}$$

Use the integral function
on a calculator to evaluate

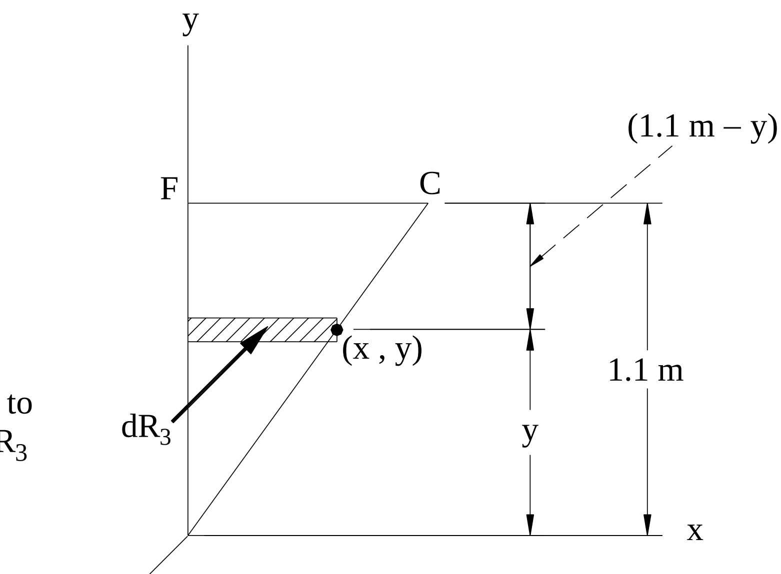
$$= 0.1109\rho g \qquad (2)$$

8 We also need to determine the line of action of R_3. We can do this by equating the moment produced by R_3 acting along its line of action to the integral of the moment produced by dR_3.

9 Moment of R_3 about FC



11 Moment of differential force about FC



12 Equating moments about the top edge FC gives

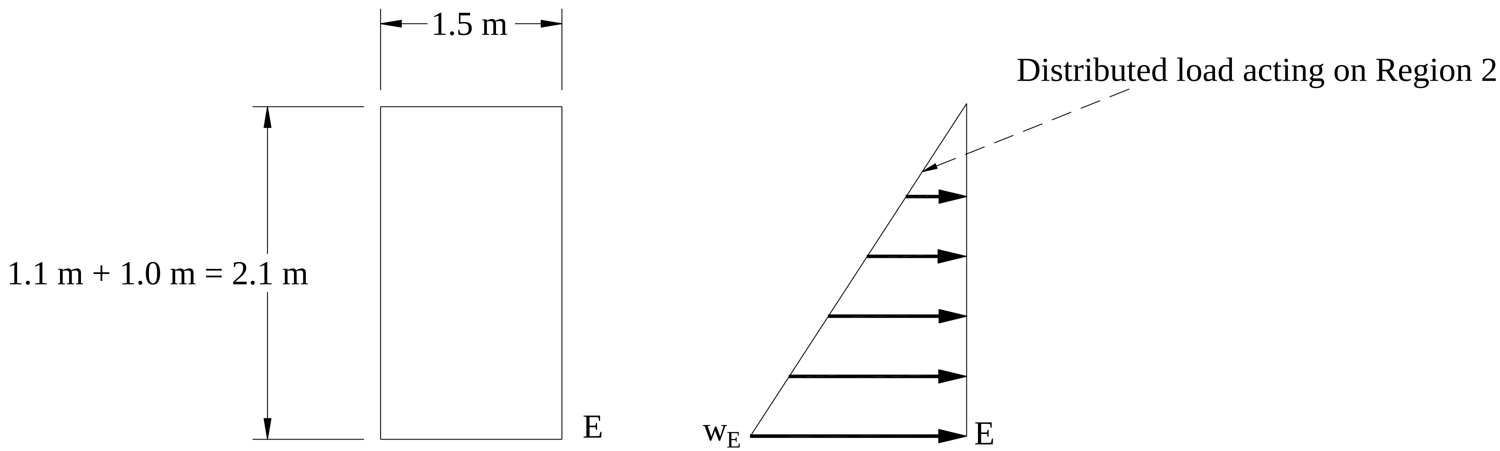
$$\underbrace{R_3}_{0.1109\rho g \text{ by Eq. 2}} \times d_3 = \int (1.1 - y)\, dR_3$$

$$= \int_0^{1.1} (1.1 - y)\, \overbrace{[\rho g(1.1 - y)\,(y/2)\, dy]}^{\text{Eq. 1}}$$

Evaluating the integral and solving for d_3 gives

$$d_3 = 0.5501 \text{ m} \qquad (3)$$

13 Resultant force acting on Region 2



Pressure at point E = (2.1 m)ρg

Distributed force (force per length), w_E = pressure × width

$= 2.1\rho g \times 1.5$ m

$= 3.15\rho g$

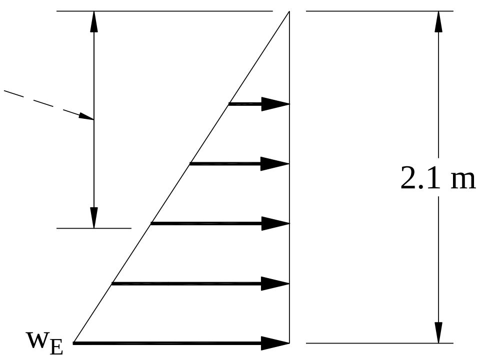
14 Resultant force

$R_2 = (1/2)(2.1 \text{ m})(3.15\rho g)$

$= 3.3075\rho g$ (4)

15 Line of action of R_2 passes through centroid of triangle.

$d_2 = (2/3)(2.1) = 1.4$ m (5)



16 Total force acting on end of tank

$R = R_1 + R_2 + R_3$

$= R_3$, by symmetry

$= R_2 + 2R_3$

by Eq. 4

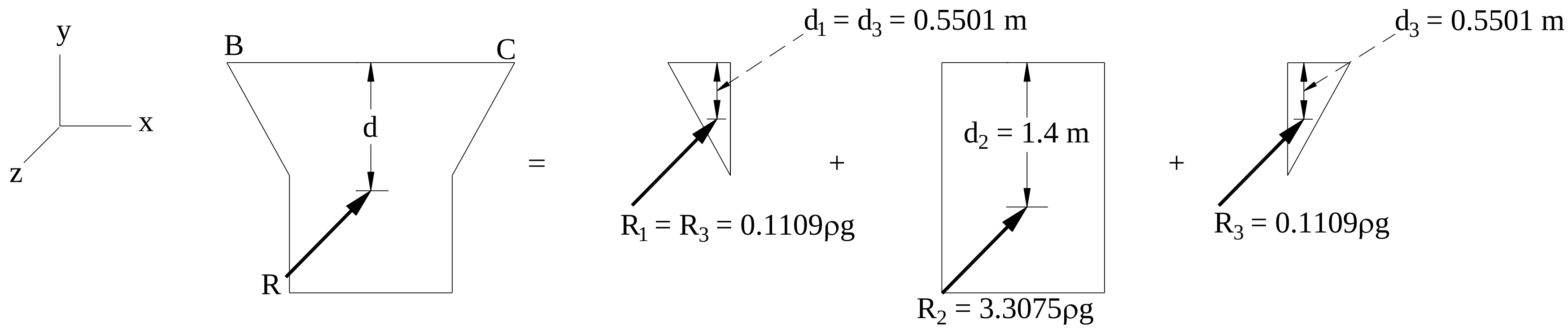
by Eq. 2

$= 3.3075\rho g + 2(0.1109\rho g)$

$= 3.5293\rho g$ (6)

Using $\rho = 10^3$ kg/m^3 and $g = 9.81$ m/s^2 gives

$R = 34.6$ kN ←Ans.



17 Determine the line of action of the resultant R by equating the moment of R to the sum of moments of R_1, R_2, and R_3:

$$\Sigma M_{BC}: Rd = R_1 d_1 + R_2 d_2 + R_3 d_3$$

Substituting the R, R_1, d_1, R_2, d_2, R_3, and d_3 values from the figures above into this equation and solving gives

$$d = 1.347 \text{ m} \qquad \leftarrow \text{Ans.}$$